MERITS OF USING INNER ACCURACY THEORY IN ADJUSTING FIRST ORDER LEVELLING NETWORKS AND DEFICIENCY OF USING APPROXIMATE ADJUSTMENT IN SECTIONS

BY

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يهدف هذا البحث الى تقديم طريقة ضبط الشبكات الحره وتطبيقها على ضبط حلقات الميزانية وذلك لتخليص قيم التباين من تأثير تثبيت أى نقط اختيارية فى حلقات الميزانية

وكذلك لتوضيح تأثير تقسيم الحلقات الخاصة بالميزانية الى قطاعات وحل كل قطاع على حده على قيم المناسيب المحددة .

Abstract

Free net adjustment technique applied to levelling networks is presented here to show the effect of librating the variances of the adjusted elevations from the arbitrary fixational parameters. The results are compared with those obtained from the classical method of adjustment with constraints. Also the effect of adjusting these nets in sections on the adjusted elevations is investigated.

1- Introduction

During the adjustment of levelling nets by the methode of observation equations, at least the elevation of one bench mark is held fixed to avoid singularity in the system of normal equations. Introducing this minimum number of constraints will have a direct effect on the values of the variances of the adjusted elevations. In order to filter out the effect of introducing certain constraints from the variance-covariance matrix of the adjusted elevations, inner accuracy theory is applied [2].

Inner accuracy theory was first introduced in 1962 by p.meissl, it allows us to filter the effect of an arbitrary set of parameters out of a given covariance matrix. the accuracy remaining after filtering is called inner accuracy [2]. In geodesy the filtering parameters most of the time are restricted to shifts, rotations and scale factor. Accordingly the inner accuracy is the accuracy being librated from the effect of these constraints. Inner accuracy theory or free net adjustment means allowing the stations to move freely without imposed constraints.

In the past years, a number of papers were devoted to the problem of free networks in which the mathematical method sought would satisfy that the sum of the squares of the standard deviations of the coordinates will be minimum . some of these papers [3], [8], were based on the generalized matrix algebra. Other use certain constraints in the solutions [6], [7]. A straight forward solution of this problem will be used here and is known as the modified observation equations [1]. This solution can be easly applied using the programs designed to handle the observation equation method without constraints.

At the same time, the effect of dividing a levelling network into sections and adjusting each section separatly by constraining the elevations obtained from previous adjustment is investigated.

2- Mathematical model

The observation equations used for adjusting the heights of a levelling net can be given in matrix form as

$$ {}_{m}A_{n} \, {}_{n}X_{1} - {}_{m}L_{1} = {}_{m}V_{1} \qquad (1) $$

Where

A = coefficient matrix of obsrvation equations

X = Unknown correction vector

L = vector of misclosure

V = Unknown vector of residuals

m = Number of observations

n = Number of Unknown parameters

The final form of this system of observation equations which satisfy the minimum condition $\Sigma v^T p v = \min$, [4], will take the form

$$(A^T p A)^{-1} X = A^T p L \qquad (2)$$

$$N . X = A^T p L$$

Where

N = Normal equation matrix

P = weight matrix

The normal equation matrix N is usually has a rank deficiency depending on the type of the problem , that is why certain parameters are eliminated, or constrained to avoid the singularity during the solution of this system.The variance covariance matrix of the adjusted elevations in this case will take the following form

$$Q_{xx} = N^{-1} \qquad (3)$$

If free net technique is applied to levelling nets,then the rank deficiency of the normal equations will be one and will be denoted as d . Accordingly the solution of this system is based on forming a transformation matrix G which satisfies the condition

$$_{d.n}G^T \quad _{n}X_{1} = 0 \qquad (4)$$

or

$$_{m}A_{n} \quad _{n}G_{d} = 0 \qquad (5)$$

In case of levelling nets the G matrix will be easily formed according to the definition of equations (4) and (5) as follows

$$G^T = \frac{1}{\sqrt{n}} [\, 1 \quad 1 \quad 1 \quad . \quad . \quad . \quad 1 \,] \qquad (6)$$

Which satisfies also the condition that

$${}_dG^T_n \; {}_nG_d = {}_dI_d \qquad (7)$$

Now the modified form , [1],of the observation equations given in equation (1) will take the form

$$\bar{A} X - \bar{L} = \bar{V} \qquad (8)$$

Where

$$\bar{A} = \begin{bmatrix} A \\ G^T \end{bmatrix} \qquad \bar{L} = \begin{bmatrix} L \\ 0 \end{bmatrix} \qquad (9)$$

the normal equations produced from the above system of equations are positive definite and the system to be solved according to the minimum principle $\Sigma\, v^T p\, v = \min$ will take the form

$$X = (\bar{A}^T P \bar{A})^{-1} \bar{A}^T P \bar{L} \qquad (10)$$

where

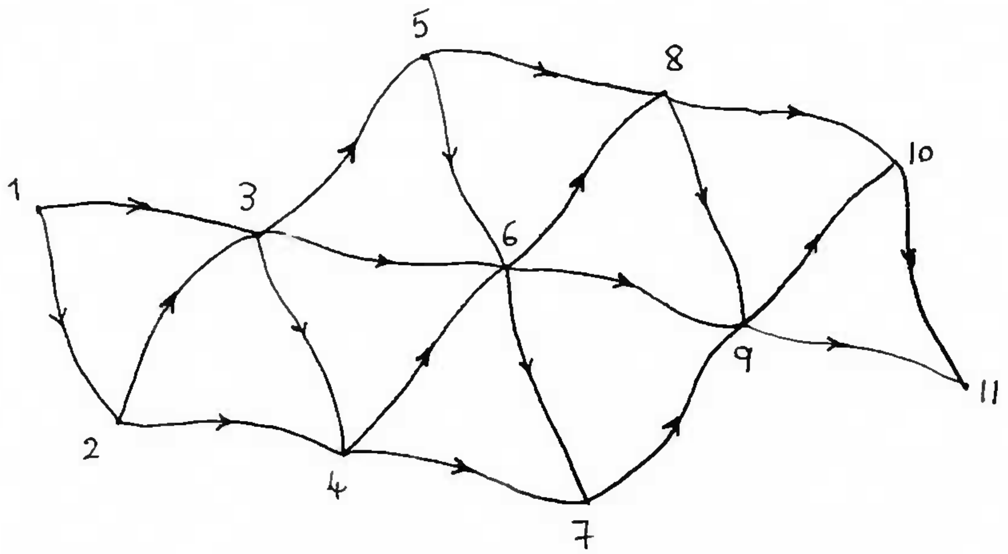
$$\bar{A}^T P \bar{A} = N + G \; G^T$$

From the above formulation one can simply use the programs designed to handle the observation equations method without constraints for solving this problem. on the other hand the variance covariance matrix of the adjusted coordinates are calculated from the following

$$Q_{xx} = (N + G \; G^T)^{-1} - G \; G^T \qquad (10)$$

3- Data under consideration

In order to apply the free net technique on levelling nets and test the effect of dividing the net into sections ,according to the schedule of work ,on the final adjusted elevations, a simulated levelling net, fig (1),consisting of eleven points with 20 elevation differences was choosen. Table (1) shows the observed elevation differences together with their lenghts .



levelling network under considerations

Fig (1)

from	to	elev.dif. m	length km	from	to	elev.diff. m	length km
1	2	.99	2	1	3	1.99	2
2	3	1.01	2	2	4	1.99	2
3	4	1.01	2	3	5	2.01	2
4	6	1.99	2	3	6	3.01	2
5	8	2.99	2	4	7	2.99	2
6	8	2.01	2	5	6	0.99	2
7	9	1.99	2	6	9	3.01	2
8	9	1.01	2	7	6	-0.99	2
9	10	1.01	2	8	10	2.01	2
10	11	1.01	2	9	11	2.01	2

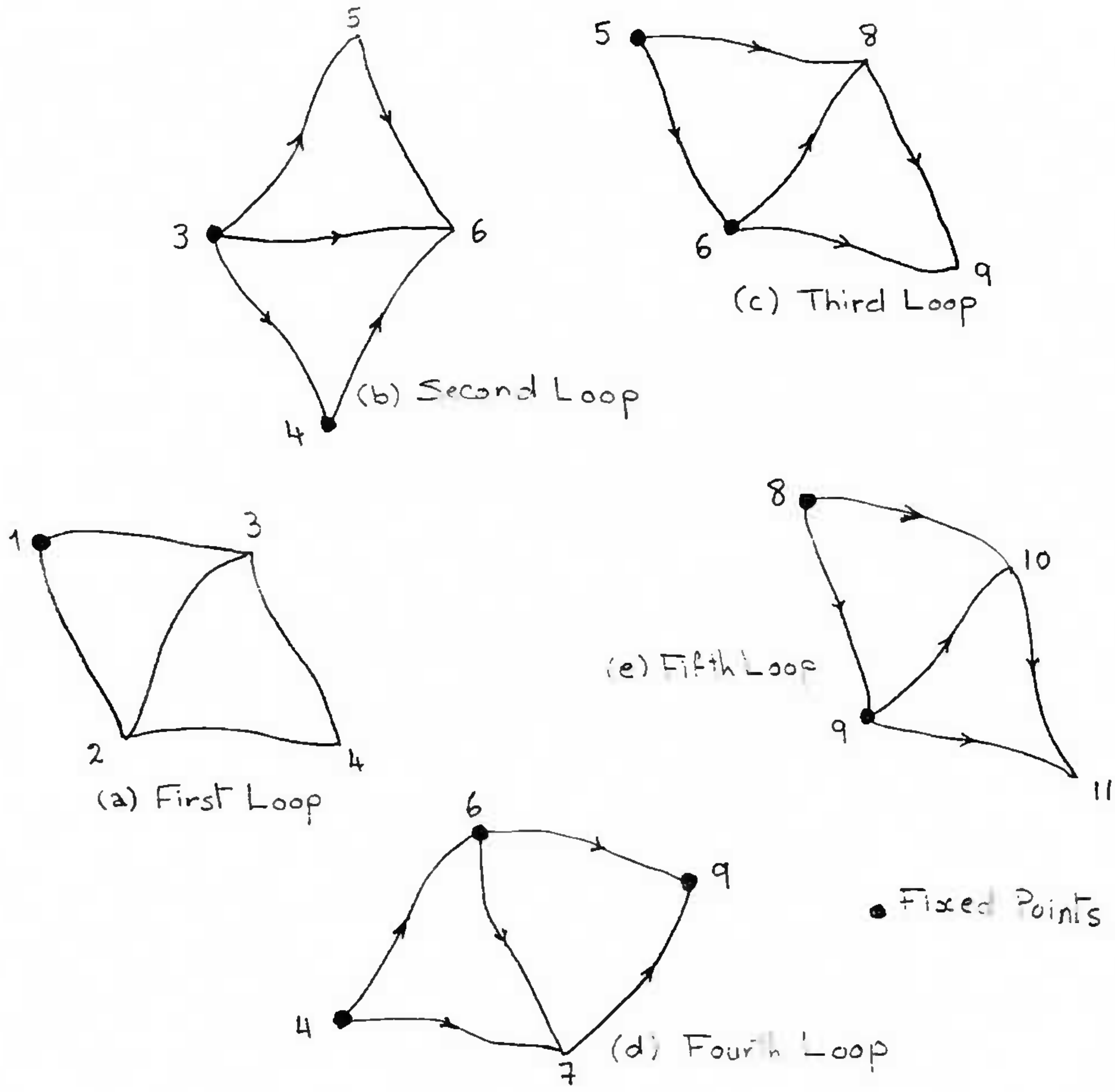
Assumed elevation differnces

Table (1)

4- Practical application and results

For the above levelling net given in fig (1) eight different adjustments were carried out . These adjustments are as follows

1) Adjusting the whole net by applying the free net technique
2) Adjusting the whole net while fixing the elevation of point 1
3) Adjusting the whole net while fixing the elevation of point 6 to the value obtained from adjustment (2).
4) Adjusting the entire net in sections as follows



Adjustment of the levelling net in successive sections

Fig (2)

a) Adjusting the first loop (1,2,3,4) while fixing the elevation of point 1 , fig (2-a).

b) Adjusting the second loop (3,4,5,6) while fixing the elevations of points (2,3) to the adjusted values obtained from adjustment (a) , fig (2-b).

c) Adjusting the third loop (5,6,7,8) while fixing the elevations of points (5,6) to the adjusted values obtained from adjustment (b) , fig (2-c).

d) Adjusting the forth loop (4,6,7,9) while fixing the elevations of points (4,6) to the adjusted values obtained from adjustments (b,c), fig (2-d).

e) Adjusting the fifth loop (8,9,10,11) while fixing the elevations of points (8,9) to the adjusted values obtained from adjustments (c,d) , fig (2-e)

The results obtained from the above adjustments are tabulated in tables (2),(3) and (4)

	adjustment 1		adjustment 2		adjustment 3	
st.	adj.elv.	st.dev.	adj.elv.	st.dev.	adj.elv.	st.dev.
1	1.00670	0.00800	1.00000	0.00000	0.99999	0.00927
2	1.99750	0.00670	1.99080	0.00763	1.99079	0.00808
3	2.99590	0.00450	2.98920	0.00763	2.98920	0.00577
4	3.99990	0.00540	3.99319	0.00889	3.99318	0.00627
5	5.00520	0.00590	4.99848	0.01012	4.99848	0.00642
6	5.99430	0.00410	5.98757	0.00927	5.98757	0.00000
7	6.99190	0.00510	6.98517	0.00951	6.98517	0.00590
8	7.99540	0.00540	7.98867	0.01059	7.98867	0.00628
9	8.99750	0.00500	8.99073	0.01051	8.99072	0.00629
10	10.00460	0.00680	9.99791	0.01182	9.99790	0.00823
11	11.01100	0.00810	11.00432	0.01271	11.00431	0.00947

Results of the first three adjustments

Table (2)

st.	adjustment a		adjustment b		adjustment c	
	adj.elv.	st.dev.	adj.elv.	st.dev	adj.elv.	st.dev.
1	1.00000	0.00000				
2	1.99000	0.00968				
3	2.99000	0.00968	2.99000	0.00000		
4	3.99000	0.01224	3.99000	0.00000		
5			5.00000	0.00774	5.00000	0.00000
6			5.99000	0.00632	5.99000	0.00000
8					7.99400	0.00346
9					9.00200	0.00424

Results of the first three loops

Table (3)

st.	adjustment d		adjustment e	
	adj.elv.	st.dev.	adj.elv.	st.dev
4	3.99000	0.00015		
6	5.99000	0.00000		
7	6.99067	0.01066		
8			7.99400	0.00000
9	9.00200	0.00000	9.00200	0.00000
10			10.00680	0.00304
11			11.01440	0.00373

Results of adjusting the last two loops in sections

Table (4)

5- Conclusions

From the above results one can conclude the following

1) The st.dev. of the adjusted elevations comming out from

applying free net technique are smaller than that of using minimum constraints, adjustment (2) or (3), which is an indication that during this adjustment the st.dev. of the points were liberated from the effect of fixations and are the the true values for the bench marks.

2) The minimum value of st.dev. from free adjustment was found at bench mark (6) which is at the geometric centre of the net.
3) The values of st.dev. from adjustment (3) are similarly behaving like the values from adjustment (1) which indicates that in order to minimize the effect of fixation on the st.dev of the elevations and to make them very near to the true values, one should choose the fixed point to be very near to the geometric centre of the levelling net to be adjusted.
4) In case free net adjustment is applied , logicaly the adjusted values of the elevations will not be tied to any desired reference, but at the same time the differences in levels between any two bench marks will be the same as in any usual rigorous adjustment.
5) When dividing the net into loops or sections and each loop is adjusted separetly, by fixing the common bench marks from the preceeding sections,certain error propagations were introduced this was very clear from tables (3) and (4).For example the elevation of point 11 was found to be 11.00432 m from adjustment (2) while by dividing the net into loops, its elevation, table (4), was found to be 11.01440 m ,with a difference of 1 cm in only five loops of an approximate length of 6 kilometrs. The defficiency of such a method, is in agreement with [5], and the rigorous methods, [4], regarding this problem should be applied.

6- Recommendations

From the above given results and conclusions the following recommendation are introduced:

1) All the first order levelling loops which were divided and adjusted in sections should be readjusted, simultaneously taking into consideration that all the loops will be adjusted in one step.
2) If the levelling net is too big to be solved in one block then dividing the net into sections and solving each section separately as shown here is not recommended, and the rigorous methods should be applied.
3) In order to liberate the st.dev. of the adjusted elevations from the effect of the fixiational parameters, either free net technique , or choosing a point very near to the geometric centre of the net and dealing with it as a fixed point in the classical adjustment is recommended to be applied.
4) If free net technique is used then after the free adjustment is completed then any bench mark can be taken as a datum and the required elevation is assumed to it as a correction and at the same time all the elevations of the other points are given the same coorection applied to the bench mark which was taken as a datum.

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